AHSMC 2010 Unofficial Solutions

HWW Math Club 11/22/2010

1. How many positive integers n are there such that 4n has exactly 2 digits?

2. A 4x6 plot of land is divided into 1x1 lots; what is the total length of the fencing?

3. The GCD of two numbers is 1 and the LCM is 10; what is their sum?

4. How many solutions (non-negative integers) to 3x+2y=27?

5. In the sequence 1,2,3,4,6,7,8,9,... what is the 2010th term?

6. 5 people in a hotel are on floors 1,2,3,21,40; what floor should they meet to minimize total travel distance?

7. Two pigeons randomly enter a 3x3 coop; what's the probability they are on opposite sides of an interior wall?

8. For what values of x is it that $\frac{1}{x} \leq -3 \leq x$?

9. Quadrilateral ABCD; AB||DC, DC=2AB, ADC=30, BCD=50; M is the midpoint of CD. What is AMB?

10. How many isosceles but non-equilateral triangles are there with integer sides between 1 and 9 inclusive?

11. Which is the largest (towers of powers)?

22223   22232   22322   23222   32222

12. A gold number is a positive integer with the form ab+a+b (a,b positive integers). How many gold numbers between 1 and 20 inclusive?

13. Tetrahedron: DA, DB, DC perpendicular. DA=1, DB=DC=2, then what is the radius of the sphere passing through A,B,C,D?

14. $f(x) = x^2$ and $g(x) = x^4$; applying f 50 times and g 49 times gives x^n, find n.

15. ABC has area 1. X, Y are points on AB and Z on AC such that XY=2AX, XZ||YC, YZ||BC. What's the area of XYZ?

16. How many n (integers) where $2n + 1 \mid n^3 - 3n + 2$?

What did you get on the AHSMC?